Proof by induction

Three steps for proof by induction.

1. verify that inital condition satisfy the statement.
2. Prove that when n satisfies the statement, $n + 1$ also satisfies.
3. Two Sentences

The two sentences you must write when the statement is proved are:

"if the result is true for $n = k$, then it's true for $n = k + 1$"

"Since it's true for $n = 1$, it is true for *(condition)*, *(statement)*"

(fill in the condition and statement yourself)

Only three types of induction questions have appeared in the exam.

1. Series

e.g: A sequence is defined by $u_{n+1} = 4u_n - 3, u_1 = 2$. Prove that $u_n = 4^{n-1} + 1$ for $n \in \mathbb{Z}^+$

solution:

for $n = 1, u_1 = 4^0 + 1 = 2$

Assume the result is true for $n = k$, for $n = k + 1$

$u_{k+1} = 4u_k - 3 = 4\left(4^{k-1} + 1\right) - 3 = 4^k + 1$

if the result is true for $n = k$, then it's true for $n = k + 1$ Since it's true for $n = 1$, it is true for all $n \in \mathbb{Z}^+$, $u_n = 4^{n-1} + 1$

2. Derivatives

e.g Prove for every integer $n \geq 3$

$$\frac{\mathrm{d}^n y}{\mathrm{d}x^n} = (-1)^{n-1} \frac{2(n-3)!}{(1+x)^{n-2}}$$

3. Matrix

e.g Prove that for $n \in \mathbb{Z}^+$

$$\begin{pmatrix} 1 & 0 \\ -1 & 5 \end{pmatrix}^n = \begin{pmatrix} 1 & 0 \\ -\frac{1}{4}(5^n - 1) & 5^n \end{pmatrix}$$