Vectors

3 Equation of a straight line.

1.

$$\boldsymbol{r} = \begin{pmatrix} a \\ b \\ c \end{pmatrix} + \lambda \begin{pmatrix} d \\ e \\ f \end{pmatrix}$$

2.

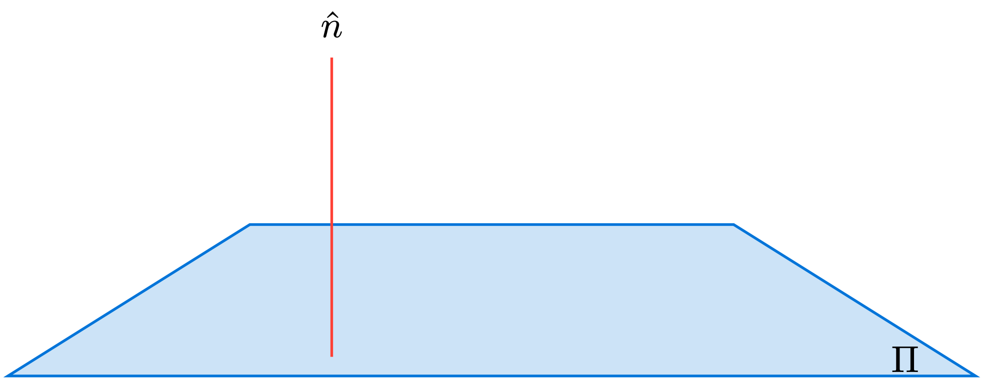
$$\frac{x-a}{d} = \frac{y-b}{e} = \frac{z-c}{f} \ (= \lambda)$$

3.

$$\begin{cases} x = a + d\lambda \\ y = b + e\lambda \\ z = c + f\lambda \end{cases}$$

Plane & Cross product

A plane Π can be define be defined by a point on the plane $M(x_0, y_0, z_0)$ and a normal vector $\hat{n} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$



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The plane can be expressed in cartesian form as

$$a(x - x_0) + b(y - y_0) + c(z - z_0) = 0$$

consider $d = -ax_0 - by_0 - cz_0$ we can derive the standard equation of the plane.

$$ax + by + cz = d$$

Note: Hereby we can infer that the normal vector to the plane is $\hat{n} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}$

Calculate cross product

Cross product definition: $|\boldsymbol{a} \times \boldsymbol{b}| = |\boldsymbol{a}||\boldsymbol{b}| \sin \theta$

calculate cross product: consider $\boldsymbol{a} = \begin{pmatrix} a \\ b \\ c \end{pmatrix}, \boldsymbol{b} = \begin{pmatrix} d \\ e \\ f \end{pmatrix}$,then.

$$\hat{n} = \boldsymbol{a} \times \boldsymbol{b} = \begin{vmatrix} \boldsymbol{i} & \boldsymbol{j} & \boldsymbol{k} \\ a & b & c \\ d & e & f \end{vmatrix}$$

The result $\hat{n}$ is prependicular to any vector on the plane.

Use three noncollinear points to describe a plane.

If there are three noncollinear points $M_0 = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}, M_1 = \begin{pmatrix} x_2 \\ y_2 \\ z_2 \end{pmatrix}, M_2 = \begin{pmatrix} x_3 \\ y_3 \\ z_3 \end{pmatrix}$

Then they can represent a plane Π which its cartesian form is.

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ x_2 - x_1 & y_2 - y_1 & z_2 - z_1 \\ x_3 - x_1 & y_3 - y_1 & z_3 - z_1 \end{vmatrix} = 0$$

Alternatively, given two noncollinear vectors $\hat{a} = \begin{pmatrix} a_1 \\ a_2 \\ a_3 \end{pmatrix}, \hat{b} = \begin{pmatrix} b_1 \\ b_2 \\ b_3 \end{pmatrix}$ and a point $P = \begin{pmatrix} x_1 \\ y_1 \\ z_1 \end{pmatrix}$, the plane Π can be expressed as

$$\begin{vmatrix} x - x_1 & y - y_1 & z - z_1 \\ a_1 & a_2 & a_3 \\ b_1 & b_2 & b_3 \end{vmatrix} = 0$$

Transform Cartesian form to other 2 forms of plane

1. Scalar form ($\hat{r} \cdot \hat{n} = \hat{a} \cdot \hat{n}$)

for $Ax + By + Cz = d$, $\hat{n} = \begin{pmatrix} A \\ B \\ C \end{pmatrix}$

The RHS equals to d

therefore $\hat{r} \cdot \hat{n} = d$

2. Vector form. ($\hat{r} = \hat{a} + \lambda\hat{b} + \mu\hat{c}$)

for $Ax + By + Cz = d$, randomly choose 3 points A, B, C on the plane, for example $x = y = 0, y = z = 0, x = z = 0$.

Then the equation can be formed by

$$\hat{r} = \overrightarrow{\mathrm{OA}} + \lambda\overrightarrow{\mathrm{AB}} + \mu\overrightarrow{\mathrm{AC}}$$

Calculate distance

1. The distance of a point $P(x_0, y_0, z_0)$ from a plane $\Pi : Ax + By + Cz = d$ can be calculated using.

$$\frac{|Ax_0 + By_0 + Cz_0 - d|}{\sqrt{A^2 + B^2 + C^2}}$$

2. let a straight line be $\hat{r} = \hat{a} + \lambda\hat{b}$. let $\overrightarrow{\mathrm{OA}} = \hat{a}$, the distance of a point $P(x_0, y_0, z_0)$ can be calculated using

$$\frac{\left|\overrightarrow{\mathrm{AP}} \times \hat{b}\right|}{\left|\hat{b}\right|}$$

3. Consider $\hat{r}_1 = \hat{a}_1 + \lambda\hat{b}_1$ and $\hat{r}_2 = \hat{a}_2 + \mu\hat{b}_2$, their distance were given by

$$\left|\frac{\hat{b}_1 \times \hat{b}_2}{\left|\hat{b}_1 \times \hat{b}_2\right|} \cdot (\hat{a}_1 - \hat{a}_2)\right|$$

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